Stress Testing of Credit Migration
A Macroeconomic Approach

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Table of Contents

Executive Summary 3

Historical Data Analysis 7
  Ratings maintaining status 8
  Ratings changing status 8

The Model 11
  Case 1: One-Notch Transitions 11
  Principal components 11
  Estimation of one-notch movements using the one-notch components as drivers 16
  Case 2: Two-Notch Transitions 17
  Approach 1 – Zero/non-zero series 17
  Approach 2 – Non-zero series 18

Appendix A – One-Notch Downgrades 20
Appendix B – One-Notch Upgrades 24
Appendix C – Two-Notch-Plus Downgrades 27
Appendix D – Two-Notch-Plus Upgrades 34
Appendix E – In-Sample Fitted Values 37

About the Authors 39
Executive Summary

The main purpose of our exercise is to stress-test the elements of a standard, through-the-cycle rating transition matrix with an explicit and transparent connection to macroeconomic drivers. The challenge behind this exercise is evident in the time-series nature of through-the-cycle migrations: They are built to be stable over time, and changes happen in waves (of upgrades or downgrades) that disappear once the economic and credit conditions go back to normal. In other words, their behaviour over time is not symmetric around an average or median value. They actually show bimodal distributions, with observations accumulated in different states of nature: (i) normal and/or good times vs. (ii) stressed conditions.

Executive Summary Figure 1: Bimodal Nature of Credit Transitions
Bimodal distribution of Baa to Ba credit migrations (bar chart) versus a normal symmetric distribution (solid line)

If the modeller attempts to analyze these series with standard time-series tools, the estimations will not be efficient and could even be biased. Standard linear models will attempt to fit a symmetric curve whose median/mean is not representative of the underlying data process. This is illustrated by adding a normal-shape curve to the histogram in Figure I. The residuals from such an estimation process will be large and the model-fit poor (very low R2, for instance).

What we propose in this article is to handle these states of nature through quantile or discrete-choice methods in order to properly model the probability of the system to face stressed versus normal periods. This probability correlates strongly with macroeconomic drivers and allows the analyst to perform sound macroeconomic stress testing exercises to through-the-cycle credit migrations.
Implementing a two-stage model approach

Step 1: Discrete choice model

This step helps the modeller identify the probability of the system to be at a particular state of nature (for example, stressed condition versus normal/good state, zero versus non-zero migrations).

A probit model is formulated for the probability of several discrete states of nature: 

\[ p(state_1, state_2) > 0 \]

The specific formulation of our probit regression is:

\[ P(d_t = 1|X_{t,t+1}, \beta) = F(X_{t,t+1}, \beta) \]

where \( d_t \) is a dummy variable that indicates when the state of nature 1 occurs. For example: a non-zero two-notch movement (upgrade or downgrade), \( X_{t,t+1} \) is a vector of explanatory variables, including macroeconomic and credit (point-in-time PDs) drivers, \( \beta \) is the vector of parameters to be estimated, and \( F \) is the cumulative distribution function.
Table I: Binary (Probit) Estimation Output on a One-Notch Upgrade Component, With Macro Drivers

Probit regression

Number of obs = 141
LR chi2(15) = 130.41
Prob > chi2 = 0.0000
Log likelihood = -29.926872 Pseudo R2 = 0.6854

|                      | Coef.  | Std. Err. | z     | P>|z|     | [95% Conf. Interval] |
|----------------------|--------|-----------|-------|---------|----------------------|
|                      | d_notch1_upg_comp |         |       |         |                      |
|                      | tyyxxgdpus_cust   | 48.80328 | 19.24714 | 2.54 | 0.011 | 11.07959, 86.52698   |
|                      | F6.         | 21.43659 | 26.64552 | 0.80 | 0.421 | -30.78766, 73.66084  |
|                      | tyyxxgdpeuro_cust| 134.45   | 38.7185 | 3.47 | 0.001 | 58.56311, 210.3368   |
|                      | F12.        |          |         |       |         |                      |
|                      | usa_lbr_cust   | -.2693973| .1235558| -2.18 | 0.029 | -.5115622, -.0272323 |
|                      | F12.        |          |         |       |         |                      |
|                      | seas_mth1     | -.075104 | .9107124| -0.08 | 0.934 | -.860068, 1.70986    |
|                      | seas_mth2     | -1.056447| .9021973| -1.12 | 0.267 | -1.873919, 1.66263   |
|                      | seas_mth3     | -.04603  | .9028297| -0.05 | 0.959 | -1.815544, 1.723484  |
|                      | seas_mth4     | .0487501 | .910295 | 0.05  | 0.957 | -1.735395, 1.832896  |
|                      | seas_mth5     | .1317361 | .9128368| 0.14  | 0.885 | -1.657391, 1.920863  |
|                      | seas_mth6     | 1.06986  | .9774743| 1.09  | 0.274 | -1.845954, 2.985674  |
|                      | seas_mth7     | .0534994 | .8965134| 0.06  | 0.952 | -1.703635, 1.810633  |
|                      | seas_mth8     | .2468421 | .8945587| 0.28  | 0.783 | -1.506461, 2.000145  |
|                      | seas_mth9     | -.3058699| .9388028| -0.33 | 0.745 | -2.14589, 1.53415    |
|                      | seas_mth10    | -.1047082| .9450209| -0.11 | 0.912 | -1.956915, 1.747499  |
|                      | seas_mth11    | .0102609 | .9382816| 0.01  | 0.991 | -1.828737, 1.849259  |
|                      | seas_mth12    | 0 (omitted) |         |       |         |                      |
|                      | _cons        | -2.545835 | 1.326153 | -1.92 | 0.055 | -5.145046, .0533769  |

Executive Summary Figure 3: Binary (Probit) Regression on the Upgrade 0-1 Median Variable

Historical data (actuals and fitted), two states of nature: (1) higher than or (2) lower than median

![Executive Summary Figure 3: Binary (Probit) Regression on the Upgrade 0-1 Median Variable](image-url)
Step 2: Conditional estimation

We estimate a regression of the variable itself (for example, the probability of a two-notch transition from any given bucket) on the list of potential exogenous drivers (for example, one-notch transitions, macroeconomic drivers) conditional on the variable of interest being positive: \( p[y_1, y_2 | d_t = 1] = f(X_{t,t+1}, \theta) \), where \( f \) is a linear function on the explanatory vector, \( X_{t,t+1} \), and a vector of parameters, \( \theta \). The final prediction is obtained through the joint probability of the two steps.

Executive Summary Figures IV and V:

Ba to Default and CaaC to Default Transitions, Modelled Using the Two-Step Process

Historical data (actuals and fitted), predictions under Baseline, FSA Anchor, Severe Scenario 4, and Custom Scenarios
Historical Data Analysis

The shifts of global corporates between bond credit ratings have been collated over 12-month transitions for every month from January 1990 to September 2011. The ratings are grouped into the following categories to improve the transition stability: { Aaa, Aa, A, Baa, Ba, B, CaaC (incorporating Caa, Ca and C), DEF, WR—Rating withdrawn }.

This paper looks to model the movement between bond credit ratings using economic indicators to account for the variance across time. The typical rating transition matrix that we look at includes information on the probability of any rating class keeping its current status or being downgraded (upgraded), in any magnitude, 12 months from the current period. We have collected monthly data for all the ratings grouped in each bucket listed above, for all companies globally. Three types can be identified in the behavior of the different notches in a rating matrix: ratings that keep their status (elements in the diagonal), ratings that are downgraded or upgraded by only one notch (elements right next to the diagonal), and ratings that are downgraded or upgraded by two or more notches.

In what follows, two types of notations will be employed: The symbol $x_y$ represents the probability that a bucket rated $x$ on a given date will be rated $y$ a year from that date. Such probability can also be noted as $p(b, b+\text{n})$, where $b$ is any given rating bucket and $n$ is the number of notches that it moves.

Tables 1 and 2 summarize the behavior of the ratings in two time periods. The first window, January 1983 to January 2007, focuses on a longer period with several global economic cycles in it. The second window, June 2007 to October 2009, captures the action in rating transitions in the convoluted times of the financial crisis of the last few years. As expected, the probability of remaining highest quality (Aaa) dramatically decreases in times of severe disruptions in the economy. Also, the probabilities along the main diagonal drop almost monotonically. The only exceptions are those of Baa and CaaC buckets, which experience a slight rise in exchange for lower probabilities of upgrades and higher probabilities of defaults, along the same row. Finally, the notches on the secondary diagonals also behave consistently: The lower-left elements (upgrades) present significantly lower probabilities, while the upper-right elements (downgrades) exhibit significantly higher probabilities, including those notches going to default.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-c</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aaa</strong></td>
<td>92.10%</td>
<td>7.52%</td>
<td>0.33%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td><strong>Aa</strong></td>
<td>0.99%</td>
<td>90.49%</td>
<td>8.07%</td>
<td>0.37%</td>
<td>0.04%</td>
<td>0.03%</td>
<td>0.00%</td>
<td>0.02%</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>0.07%</td>
<td>2.76%</td>
<td>90.65%</td>
<td>5.67%</td>
<td>0.65%</td>
<td>0.15%</td>
<td>0.03%</td>
<td>0.02%</td>
</tr>
<tr>
<td><strong>Baa</strong></td>
<td>0.05%</td>
<td>0.24%</td>
<td>5.51%</td>
<td>87.91%</td>
<td>4.75%</td>
<td>1.14%</td>
<td>0.23%</td>
<td>0.17%</td>
</tr>
<tr>
<td><strong>Ba</strong></td>
<td>0.01%</td>
<td>0.07%</td>
<td>0.47%</td>
<td>6.35%</td>
<td>82.56%</td>
<td>8.60%</td>
<td>0.60%</td>
<td>1.33%</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0.01%</td>
<td>0.05%</td>
<td>0.18%</td>
<td>0.52%</td>
<td>5.52%</td>
<td>82.90%</td>
<td>4.74%</td>
<td>6.08%</td>
</tr>
<tr>
<td><strong>Caa-c</strong></td>
<td>0.00%</td>
<td>0.02%</td>
<td>0.10%</td>
<td>1.20%</td>
<td>1.19%</td>
<td>7.12%</td>
<td>69.42%</td>
<td>20.96%</td>
</tr>
</tbody>
</table>

1 For reasons including: debt maturity, calls, puts, conversions, etc., or business reasons (for example, change in the size of a debt issue), or the issuer defaults.
Table 2
Average probabilities (2007M6 - 2009M10)

<table>
<thead>
<tr>
<th></th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-c</th>
<th>Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>78.15%</td>
<td>21.71%</td>
<td>0.04%</td>
<td>0.11%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Aa</td>
<td>0.05%</td>
<td>82.65%</td>
<td>16.03%</td>
<td>0.99%</td>
<td>0.11%</td>
<td>0.02%</td>
<td>0.07%</td>
<td>0.09%</td>
</tr>
<tr>
<td>A</td>
<td>0.00%</td>
<td>0.88%</td>
<td>89.58%</td>
<td>8.24%</td>
<td>0.44%</td>
<td>0.30%</td>
<td>0.15%</td>
<td>0.41%</td>
</tr>
<tr>
<td>Baa</td>
<td>0.01%</td>
<td>0.14%</td>
<td>2.20%</td>
<td>91.95%</td>
<td>4.40%</td>
<td>0.72%</td>
<td>0.20%</td>
<td>0.38%</td>
</tr>
<tr>
<td>Ba</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>5.10%</td>
<td>81.25%</td>
<td>10.46%</td>
<td>1.83%</td>
<td>1.32%</td>
</tr>
<tr>
<td>B</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.07%</td>
<td>0.17%</td>
<td>3.35%</td>
<td>78.31%</td>
<td>13.55%</td>
<td>4.55%</td>
</tr>
<tr>
<td>Caa-c</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.14%</td>
<td>0.23%</td>
<td>5.74%</td>
<td>71.19%</td>
<td>22.70%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Ratings maintaining status

A slightly different magnitude is present in the ratings in the range Aaa to A with respect to those between Baa and B. For instance, Aaa-Aaa oscillates around 92%, while B-B does so around 82%. Thus, the two ranges are presented in different graphs. It can be observed how the ratings generally suffered, in different degrees, through the crises of 1998, 2001 and 2008. Interestingly, a qualitatively different behavior can be identified between the ratings in the A-range and those in the B-range. If we look at Aaa, Aa and A ratings, the probability of higher ratings maintaining their status is, on average, higher in normal times. In tough times, however, higher ratings (especially Aaa) are more likely to lose their top quality than lower ratings, which are already slightly poorer quality. Consequently, the probability of Aaa ratings staying Aaa is more volatile. The case of Baa to B ratings is almost the opposite. The higher-quality bucket is significantly less volatile than the lower ratings. Since they are less solid than their counterparts Aa and A, the probability of their losing the grade is very sensitive to economic downturns.

![Ratings Maintaining Status (Diagonal)](image)

Ratings changing status

Figures 1 to 4 show upgrade and downgrade transitions split by whether the transitions are from investment grades. The figures show there are points in time where all transitions are impacted. This syncretic behavior suggests that the similarities between the transitions can be isolated using principal component analysis.
Figure 1: One-Notch Downgrade Transitions From Investment Grades

Figure 2: One-Notch Downgrade Transitions From Non-Investment Grades
Figure 3: One-Notch Upgrade Transitions From Investment Grades

Figure 4: One-Notch Upgrade Transitions From Non-Investment Grades
The Model

The different behavior of the various rating buckets requires a separate analysis. Since those buckets that register little action are those in the diagonal, our approach is to build models for one-notch and two-notch (or more) movements and make the diagonal elements just the residual probability after the rest of the notches in their rows. Because of the nature of corporates and how they are organized (industries, sectors, etc.), they tend to be highly interdependent and sensitive to the business cycle, thus generating the so-called waves of downgrades and upgrades. This is especially the case for one-notch transitions for which, at least from a theoretical viewpoint, much information could be extracted from modeling them as interdependent variables. The fact of an A rating being downgraded would plausibly incorporate information about the probability of a Baa rating being upgraded. One would suspect that the same logic also applies to other buckets not so directly related but that also involve one-notch transitions. This conjecture seems to be supported to some extent by the apparently strong interrelationship between one-notch movements in the data, as has been discussed previously. Therefore, estimating a structural vector autoregressive system seemed a suitable framework for these buckets.

The case of two-notch (or more) movements present a much different structure where all the observations are either zero or a small positive value. Also, two other features are quite interesting and require attention. On the one hand, these probabilities move by intervals; on the other hand, the transition from an interval of zero probability to one of positive probability, and vice versa, is anything but smooth. When the probability of a rating moving up or down by two or more notches is zero, it stays there for some consecutive periods before jumping to a strictly positive probability to remain in that interval for a number of successive time periods. Regime-switching types of models prove to be more appropriate in this framework.

Case 1: One-Notch Transitions

Principal components

Tables 3 and 4 below show the principal components of both the one-notch upgrades and downgrades--analysis performed across all one-notch downgrades and upgrades. Figure 5 displays the principal component of each analysis; as expected, there is a clear negative correlation (65%) between the two components.

Table 3: PCA on One-Notch Downgrade Transitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Comp1</th>
<th>Comp2</th>
<th>Comp3</th>
<th>Comp4</th>
<th>Comp5</th>
<th>Comp6</th>
<th>Comp7</th>
</tr>
</thead>
<tbody>
<tr>
<td>aaa_aaa</td>
<td>0.4042</td>
<td>-0.0996</td>
<td>-0.2871</td>
<td>-0.0921</td>
<td>-0.0867</td>
<td>-0.8521</td>
<td>0.0472</td>
</tr>
<tr>
<td>aa_a</td>
<td>0.3553</td>
<td>-0.5828</td>
<td>0.2993</td>
<td>-0.1097</td>
<td>0.5035</td>
<td>0.1190</td>
<td>0.4060</td>
</tr>
<tr>
<td>a_baa</td>
<td>0.3937</td>
<td>-0.3367</td>
<td>0.2600</td>
<td>0.3993</td>
<td>-0.2107</td>
<td>0.0794</td>
<td>-0.6737</td>
</tr>
<tr>
<td>baa_ba</td>
<td>0.3758</td>
<td>0.2589</td>
<td>0.4927</td>
<td>-0.4933</td>
<td>-0.5195</td>
<td>0.0971</td>
<td>0.1620</td>
</tr>
<tr>
<td>ba_b</td>
<td>0.3556</td>
<td>0.5479</td>
<td>0.1630</td>
<td>0.6429</td>
<td>0.1712</td>
<td>-0.0194</td>
<td>0.3221</td>
</tr>
<tr>
<td>b_caac</td>
<td>0.3849</td>
<td>0.3809</td>
<td>-0.2303</td>
<td>-0.3969</td>
<td>0.5257</td>
<td>0.1811</td>
<td>-0.4326</td>
</tr>
<tr>
<td>caac_def</td>
<td>0.3735</td>
<td>-0.1575</td>
<td>-0.6618</td>
<td>0.0764</td>
<td>-0.3450</td>
<td>0.4592</td>
<td>0.2489</td>
</tr>
</tbody>
</table>

Table 4: PCA on One-Notch Upgrade Transitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Comp1</th>
<th>Comp2</th>
<th>Comp3</th>
<th>Comp4</th>
<th>Comp5</th>
<th>Comp6</th>
</tr>
</thead>
<tbody>
<tr>
<td>aa_aaa</td>
<td>0.3576</td>
<td>0.5968</td>
<td>-0.0189</td>
<td>0.3014</td>
<td>-0.6039</td>
<td>0.2450</td>
</tr>
<tr>
<td>a_a</td>
<td>0.3811</td>
<td>0.5204</td>
<td>-0.2557</td>
<td>-0.3740</td>
<td>0.6056</td>
<td>0.1093</td>
</tr>
<tr>
<td>baa_a</td>
<td>0.4840</td>
<td>-0.1851</td>
<td>-0.3289</td>
<td>-0.1875</td>
<td>-0.2707</td>
<td>-0.7175</td>
</tr>
<tr>
<td>ba_baa</td>
<td>0.4034</td>
<td>0.4189</td>
<td>-0.3841</td>
<td>0.5966</td>
<td>0.2576</td>
<td>0.3032</td>
</tr>
<tr>
<td>b_b</td>
<td>0.4268</td>
<td>0.4010</td>
<td>0.2878</td>
<td>-0.5365</td>
<td>-0.2225</td>
<td>0.4849</td>
</tr>
<tr>
<td>caac_b</td>
<td>0.3921</td>
<td>0.0492</td>
<td>0.7718</td>
<td>0.3006</td>
<td>0.2817</td>
<td>-0.2935</td>
</tr>
</tbody>
</table>
There are spikes in the principal component data that suggest a two-stage approach to, first, model whether the data are spiking and, second, model the trend given the first-stage outcome. A negative correlation can be observed between the downgrade and upgrade components (see figure 5). With this in mind, the model can benefit from additionally interacting the two component forecasts.

A three-stage approach is applied to forecast the principal components:

1. Binary regression on the median.
2. Quantile regressions weighting towards upper and lower quantiles.
3. Vector autoregression interacting forecasts from steps 1 and 2.

Stage 1: First, a binary (probit) regression is utilized to split the data by the median. This method helps model the shock variance observed. This step is achieved by creating a 0-1 variable that is 1 if the component value is above the component median (see figure 6).
Stage 2: After fitting a 0-1 variable on the median, a model to forecast this 0-1 variable can be constructed using economic data. The regression models for the downgrade and upgrade 0-1 median variables (see figures 7 and 8) both use U.S. GDP year-on-year growth, euro zone GDP year-on-year growth, and the U.S. unemployment rate. The economic variables used in the final model are the expectations of growth six and 12 months from the current period. This is consistent with the fact that our study considers transitions probabilities one year from the date variable.

Figures 7 and 8: Binary (Probit) Regression on the Downgrade and Upgrade 0-1 Median Variable

The second stage is to model the upper and lower quantiles split by the median using quantile regression. Therefore two forecasts are created, both using quantile regression with asymmetry parameters 0.75 for the upper quantile and 0.25 for the lower quantile.

Figure 9: One-Notch Downgrade Forecasts for Upper and Lower Quantiles
Stage 3: The two stages are combined to give the component forecast using the following formula:

\[
\text{Component forecast} = \text{Binary forecast} \times \text{Upper quantile forecast} + (1-\text{Binary forecast}) \times \text{Lower quantile forecast}
\]

Figures 10 and 11: One-Notch Downgrade and Upgrade Component Forecasts (Post Step 2)

The final stage involved in forecasting the one-notch upgrade and downgrade components is to interact the two components with each other. Figures 12 and 13 below show the forecast outputs of estimating the components employing a vector autoregression technique that uses the forecast outputs from the second stage as exogenous variables. VAR is a modeling technique that allows for lagged variables of the forecast to be drivers. The nature of the approach is such that model estimates are calculated iteratively. VAR systems, pioneered by Sims (1980), have become a standard tool for modeling interdependent variables in macroeconometrics with a variety of specifications. A p-th order VAR, hereafter VAR (p), expresses an \(n \times 1\) vector \(y_t\) as a linear function of \(p\) lags of itself and possibly exogenous variables:
\[ y_t = c + \sum_{j}^{p} \phi_j y_{t-j} + \sum_{i}^{q} \psi_i x_{t-i} + \varepsilon_t \]

where \( c \) denotes an \( n \times 1 \) vector of constants, \( \phi \) denotes an \( n \times n \) matrix of autoregressive coefficients, \( x \) represents an \( n \times 1 \) vector of exogenous variables, and \( \psi \) is an \( m \times n \) matrix of coefficients on which the exogenous variables load. Finally, \( \varepsilon \) is an \( n \times 1 \) vector of zero-mean shocks identically and independently distributed drawn from a normal distribution. In our modeling exercise, \( y_t \) includes the time series of \( i = 1, \ldots, 7 \) -notch downgrades and upgrades, whose dynamics we condition upon a set of strictly exogenous variables describing the broad business and credit cycle.

**Figures 12 and 13: Final One-Notch Downgrade and Upgrade Component Forecasts**

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\(^2\) This vector of constants can be interpreted as the averages of the processes or their long-run equilibrium values.
Estimation of one-notch movements using the one-notch components as drivers

The VAR technique is further utilized to forecast the one-notch transitions using the relevant downgrade and upgrade component forecasts as exogenous variables. One-notch transitions are grouped for the VAR modelling by whether they are downgrades or upgrades and whether the transitions are to investment grades. Subsequently, there are four VAR models for predicting all the one-notch transitions. Figures 14 and 15 show one-notch downgrade and upgrade graphs displaying the forecasts following VAR modeling. All one-notch transition graphs can be found in appendices A (downgrades) and B (upgrades), and appendices E and F display the in-sample and six-month out-of-sample model fits, respectively.

Figures 14 and 15: One-Notch Downgrade Transition Forecast From Aaa to Aa and Baa to A
Case 2: Two-Notch Transitions
We discuss in this section the treatment of transition probabilities for two or more notch movements. The modeling methodology for the rest of transitions of more than two notches follows one of two approaches driven by the similarities in transition behavior.

Approach 1 – Zero/non-zero series

For many cases, the key feature is that observations are either zero or (usually) small positive values. In fact, for a number of buckets, the zero observations are many. The shape of the data from these transitions suggests the use of discrete-choice-type of models, where the variables take one of two possible values: zero or something positive. Furthermore, this dual behavior could sensibly be captured by means of a two-step model: The first part is designed to predict the occurrence of zeroes, the second part would capture the behavior of the positive observations. This approach has the flavor of the models proposed by Heckman (1976) to correct sample-selection bias. The basic idea in his proposal is that inference based on nonrandomly selected samples of data may lead to erroneous conclusions. The classical example is that of trying to estimate the determinants of wages with observations only from those who work. Our case is slightly different in the sense that we do observe those probabilities that are not strictly positive, only that they are exactly zero. But the same modeling philosophy lies behind our approach: If you model only the positive probabilities, you lose information; you need to know both when the transition probability will be zero and, if positive, exactly what probability it will be. Thus, both parts of the data need to be modeled.

First part: A probit model is formulated for the probability of a two-notch movement occurring, \( p(b, b \pm 2) > 0 \), where \( b \) is any given rating bucket and the time period is one year. The specific formulation of our probit regression is:

\[
P(d_t = 1|X_{t,t+1}, \beta) = F(X_{t,t+1}, \beta)
\]

where \( d_t \) is a dummy variable that indicates when a two-notch movement occurs at time \( t \) (\( d_t = 1 \) if it occurs), \( X_{t,t+1} \) is a vector of explanatory variables, \( \beta \) is the vector of parameters to be estimated, and \( F \) is the cumulative distribution function of a standard normal distribution. It seems natural to include as explanatory variables the one-notch transitions that were modeled through a VAR before, \( X_{t,t+1} = (aaa_{aa}, aa_{a}, a_{ba}, bba_{ba}, ba_{b}, b_{caac}, caac_{def}, caac_{b}, b_{ba}, ba_{baa}, baa_{a}, a_{aa}, aa_{aaa}) \), where the index \( i \) denotes the possible scenarios we might be interested in using for stress testing later on, \( i \in \{\text{scenario1}, \text{scenario2}, \text{scenario3}, \ldots\} \).

Obviously, each probit model will require identifying those one-notch transitions that will be significant for each two-notch model and will finally be in the model for each part.

Second part: We estimate a regression of the variable itself (the probability of a two-notch transition from any given bucket) on relevant one-notch transitions conditional on the variable of interest being positive:

\[
p[b, b \pm 2 \mid d_t = 1] = f(X_{t,t+1}, \theta)
\]

where \( f \) is a linear function on the explanatory one-notch transitions, \( X_{t,t+1} \), and a vector of parameters, \( \theta \).

The final model for these transitions is the joint probability of the two parts. That is, if we expect any given rating bucket to be downgraded (upgraded) by two notches, we expect it to happen with certain probability. Also, notice that the outcome from the model for the first part is always a positive probability, which does not mean that \( d_t = 1 \) always. Therefore, a threshold probability needs to be identified in each case above in which we consider that \( p[b, b \pm 2] > 0 \) in the forecasting exercise. Then, we have:

\[
E[p(b, b \pm 2) \mid P(d_t = 1|X_{t,t+1}, \beta) > T] = P(d_t = 1|X_{t,t+1}, \beta) \cdot p[b, b \pm 2 \mid d_t = 1]
\]

3 Exceptions can always be modeled with standard univariate regressions. A VAR would not fit a data structure which makes it hard to detect interdependence, besides their non-negligible degree of discreteness discussed before.
\[ F(X_{t,t+1}, \beta) \cdot f(X_{t,t+1}, \theta), \quad \text{for} \quad P(d_t = 1 | X_{t,t+1}, \beta) > T \]

where \( T \) is the arbitrary threshold probability (for each case) that discriminates the future strictly positive values of the variable from those considered as zero. Notice that the explanatory variables used in the probit and regression models for each part are the one-notch movements for the different scenarios. Therefore, the projections into the future generated by the final model for any two-notch transition already incorporate the various assumptions about the economy.

**Approach 2 – Non-zero series**

Figure 16 displays two-notch downgrade transitions from Aaa and Aa. Although the transition from Aaa can be modeled using Approach 1 detailed above, the lack of zero values on the transition from Aa suggests an approach is taken similar to modeling the one-notch components.

![Figure 16: Rating Transitions From Aaa to A and Aa to Baa](image)

The transition from Aa (green line) does not have many zero values, however it does show dichotomy between values around 0.5% and 1.75%. For transitions following this behaviour, the first part of Approach 1 is followed (a binary regression model) but amended to split the data on the median instead of zero. Therefore the probability modeled is on the transition rate being greater than the median, \( p(b, b \pm 2) > \bar{x} \).

In the second part of Approach 1, a regression of the variable itself on relevant one-notch transitions is estimated conditional on the variable of interest being positive. In Approach 2, the variable of interest is always positive. Quantile regressions are estimated on the upper and lower quantiles around the median (see figures 8 and 9 for this same approach being applied to the one-notch components). The two stages are combined to give the component forecast using the following formula:

\[
\text{Transition estimation} = \text{Binary estimation} \times \text{Upper quantile estimation} + (1-\text{Binary estimation}) \times \text{Lower quantile estimation}
\]

Figure 17 shows the forecasts following the application of these techniques for Aaa to A migration. Graphs for all two-notch-plus transitions can be found in appendices C (downgrades) and D (upgrades).
All three-notch-plus upgrade transitions have insufficient historical transitions for modeling against; as a result, the conservative approach is to give their forecasts a default zero value.

Bond credit ratings can occasionally be withdrawn for reasons including: debt maturity, calls, puts, conversions or business reasons (for example, change in the size of a debt issue), or default of the issuer. Transitions to a withdrawn status are forecast using a binary regression on the 0-1 median variable followed by quantile regression, similar to non-zero three-notch-plus transitions. Graphs for all transitions to the withdrawn status can be found in appendix G.

Transitions to and from the same rating are back-populated after all other transitions are forecast. Figure 18 shows the transition to and from Aaa over the month of transition. Graphs for all other same-notch transitions can be found in appendix H.
Appendix A – One-Notch Downgrades

One-Notch Downgrade Transition Forecast From Aaa to Aa

One-Notch Downgrade Transition Forecast From Aa to A
One-Notch Downgrade Transition Forecast From A to Baa

One-Notch Downgrade Transition Forecast From Baa to Ba
One-Notch Downgrade Transition Forecast From Ba to B

One-Notch Downgrade Transition Forecast From B to CaaC
One-Notch Downgrade Transition Forecast From CaaC to Default
Appendix B – One-Notch Upgrades

One-Notch Upgrade Transition Forecast From Aa to Aaa

One-Notch Upgrade Transition Forecast From A to Aa
One-Notch Upgrade Transition Forecast From Baa to A

One-Notch Upgrade Transition Forecast From Ba to Baa
One-Notch Upgrade Transition Forecast From B to Ba

One-Notch Upgrade Transition Forecast From CaaC to B
Appendix C – Two-Notch-Plus Downgrades

Aaa to A

Aa to Baa
A to Ba

Baa to B
Baa to CaaC

Baa to Default
Ba to Default

![Chart showing transition percentages for different scenarios over time. The x-axis represents the month of transition from 2000m1 to 2016m1, and the y-axis represents transition percentages ranging from 0 to 0.03. The chart includes lines for Actuals, Baseline, FSA, Scenario 4, and Custom scenarios.]
Appendix D – Two-Notch-Plus Upgrades

A to Aaa

Baa to Aa
Ba to A

<table>
<thead>
<tr>
<th>Month of Transition</th>
<th>Actuals Baseline</th>
<th>FSA Scenario4</th>
<th>Custom</th>
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<tr>
<td>2014m1</td>
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<tr>
<td>2016m1</td>
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B to Baa

<table>
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<th>Month of Transition</th>
<th>Actuals Baseline</th>
<th>FSA Scenario4</th>
<th>Custom</th>
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</table>
CaaC to Ba

Transition %

Month of Transition

2000m1 2002m1 2004m1 2006m1 2008m1 2010m1 2012m1 2014m1 2016m1

Actuals Baseline FSA Scenario4 Custom
Appendix E – In-Sample Fitted Values

One-Notch Downgrades

Transition %

Month of Transition
One-Notch Upgrades

- aa_aaa In_Sample_Fit
- a_aa In_Sample_Fit
- baa_a In_Sample_Fit
- b_ba In_Sample_Fit
- caa_b In_Sample_Fit

Transition %

Month of Transition

2000m1 2002m1 2004m1 2006m1 2008m1 2010m1 2012m1
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